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Heavy Vehicle Pitch Dynamics and Suspension Tuning

Part I: Unconnected Suspension

Dongpu Cao*, Subhash Rakheja and Chun-Yi Su
CONCAVE Research Center, Department of Mechanical and Industrial Engineering
Concordia University, Montreal, Quebec, H3G 1M8, Canada
Emails: dongp_ca@yahoo.com; rakheja@vax2.concordia.ca; cysu@me.concordia.ca
*Corresponding author. Fax: 1-514-848-8635

Abstract: The influence of suspension tuning of passenger cars on bounce and pitch ride performance has been explored in a number of studies, while only minimal efforts have been made for establishing similar rules for heavy vehicles. This study aims to explore pitch dynamics and suspension tunings of a two-axle heavy vehicle with unconnected suspension, which could also provide valuable information for heavy vehicles with coupled suspensions. Based on a generalized pitch plane model of a two-axle heavy vehicle integrating either unconnected or coupled suspension, three dimensionless measures of suspension properties are defined and analyzed, namely the pitch margin (PM), pitch stiffness ratio (PSR) and coupled pitch stiffness ratio (CPSR), for different unconnected suspension tunings and load conditions. Dynamic responses of the vehicle with three different load conditions and five different tunings of the unconnected suspension are obtained under excitations arising from three different random road roughness conditions and a wide range of driving speeds, and braking maneuvers. The responses are evaluated in terms of performance measures related to vertical and pitch ride, dynamic tire load, suspension travel, and pitch attitude control characteristics of the vehicle. Fundamental relationships between the vehicle responses and the proposed suspension measures (PM, PSR and CPSR) are established, based on which some basic suspension tuning rules for heavy vehicles with unconnected suspensions are also proposed.

Keywords: Heavy vehicle pitch dynamics, suspension tuning, unconnected suspension, dimensionless suspension measures, tuning rules.

1. INTRODUCTION

The performance characteristics of a heavy vehicle are strongly related to its pitch motion, which include the ride, handling, suspension stroke and dynamic tire loads. This is partially attributed to the relatively large wheelbase and coupling between the vertical and pitch motions of heavy vehicles. Moreover, heavy vehicles are generally characterized by highly variable gross vehicle weight (GVW) and load distributions, compared with passenger cars. Passenger cars are generally designed to achieve front/rear load distribution ratio and dynamic index (k^2/ab) close to unity (Gillespie, 1992; Cole, 2001), where k is the radius of gyration of the sprung mass in pitch, and a and b are the longitudinal distances from the center of gravity (c.g.) to the front and rear wheel centers, respectively. These ratios are significantly different for heavy vehicles, where the dynamic index may assume a value greater than 1 for two-axle vehicles, leading to pitch mode natural frequency lower than the bounce mode frequency (Bastow, 1987). The dynamic characteristics of heavy vehicle systems are therefore considerably different from those of the passenger cars (Cole, 2001).

Suspension design of road vehicles necessities a complex compromise among different performance measures related to ride and handling qualities. Unlike automobiles, the transport productivity and efficiency are generally prioritized for heavy vehicles, particularly the directional and roll dynamic performance (Gillespie, 1985; Fancher and Balderas, 1987; Lewis and El-Gindy, 2003). The ride properties of heavy vehicles concern the preservation of health, safety and comfort of the drivers and/or passengers, and protection of the cargoes, while the suspension design is subject to the constraints imposed by requirements on productivity and functional efficiency (Gillespie, 1985). Considering the long exposure duration of professional drivers of commercial vehicles, the driver fatigue arising from ride vibration environment may also affect the safety and efficiency of vehicle operations in an adverse manner (Jiang et al., 2001). The ride comfort performance of heavy vehicles are generally dominated by the pitch plane motions, while the lateral and roll vibrations have been considered to be relatively less important (Gillespie, 1985). The heave vibration arises from combined bounce and pitch motions, while the longitudinal vibration at the driver location is predominated only by pitch motions. Furthermore, the pitch motions are generally considered 'objectionable' and annoying, as they encourage pitch oscillation of the seat backrest and thus 'head nod' (Gillespie, 1992; Qiu and Griffin, 2005). The suppression of pitch vibration is therefore very important for enhancement of ride comfort of heavy vehicles.

The vehicle braking or acceleration maneuvers induce pitch motions and longitudinal load transfers among different axles, and thus variations in normal tire load (Dahlberg, 1999; Wong, 2001). Considerably larger load transfers may be anticipated in heavy vehicles with floating cargoes, such as partly-filled tank trucks (Kang et al., 2002). Such variations in normal load can influence the longitudinal, and cornering forces, and self-aligning torque developed by a tire. The directional and braking control performance of heavy vehicles can therefore be influenced by the pitch-induced normal load variations.

The road-friendliness of heavy vehicles has been one of the important design and regulation objectives. The variations in dynamic tire load have been known to accelerate road-damage (Cebon, 1999). The road damaging potential of an articulated vehicle combination is influenced by interactions between the vehicle units, which are strongly coupled by their respective pitch motions (Cole and Cebon, 1998).

Studies on coupled bounce and pitch motions of automobiles have resulted in some guidelines on suspension design in order to achieve pitch ride control. A few studies have investigated the effects of suspension tuning for passenger cars on ride performance enhancement. The simulation results obtained for a 4-DOF pitch plane model suggested that the front/rear suspension stiffness ratio significantly affects pitch displacement responses (Crolla and King, 1999), while the well-known 'Olley's tuning' is beneficial at higher speeds (Sharp and Pilbeam, 1993; Sharp, 2002). Odhams and Cebon (2006) developed a pitch plane vehicle model with coupling between the front and rear suspensions, where the conventional unconnected suspension was shown to be a special case of coupled suspension. The study presented that pitch plane formulations leaded to a relation between the bump and pitch response of an automobile, which was similar to the concept of Static Margin used in vehicle handling analysis. The study concluded that for the unconnected suspensions, the 'Olley's tuning' provided a nearly optimal solution for minimizing horizontal acceleration at the chest. The resulting vertical chest acceleration, however, could not be considered optimal. The study also demonstrated that an interconnected suspension with lower pitch stiffness, opposed to the conventional unconnected suspension, could offer benefits for improving dynamic tire force and body acceleration responses of a passenger car.

The tuning methodologies and design guidance provided in above-mentioned studies are applicable for passenger car suspensions, and may not be valid for heavy vehicle suspensions. Similar design rules for heavy vehicles have not yet been established, which may in part be attributed to vast variations in loading conditions. Furthermore, the validity of the recommended design rules has not been investigated in view of responses under braking and acceleration inputs. Cole (2001) also pointed out that optimal suspension tunings achieved under certain driving velocities may not work well for other speeds, since heavy vehicles operate in a wide range of speeds. This assertion would also be applicable for passenger cars. Odhams and Cebon (2006) showed that minimizing the front and rear tire force responses of a passenger car yields different suspension design parameters, which would necessitate a design compromise. Moreover, control of suspension travel that influences both the ride and handling qualities of vehicles is another important design task. The minimization of suspension travel could also improve the productivity of heavy vehicles, considering the regulations on the heavy vehicle dimensions (Cole, 2001). The study also suggested that the power dissipations due to alternative suspension designs should be assessed in future studies. Carruthers (2005) stated that for a particular passenger car, the power dissipation of the suspension system is approximately 80 W and 100 W at 50 km/h and 100 km/h, respectively.

Heavy vehicle pitch-coupled suspension systems have been investigated in a few studies (Cao et al., 2006b; Cao et al., 2007). A generalized model for a class of interconnected hydro-pneumatic suspensions was developed and analyzed under braking and road roughness inputs. The results of the studies demonstrated that the pitch-connected suspension configurations could help realize tuning of suspension pitch stiffness and damping properties independent of the front and rear vertical suspension rates. The heavy vehicle integrating pitch-connected suspensions with enhanced pitch stiffness and/or damping could inhibit braking dive and improve straight-line braking performance.

Pitch dynamics and suspension tuning of a two-axle heavy vehicle with unconnected suspension are explored in this study, in an attempt to establish a relationship between the performance characteristics of the vehicle and suspension properties. The analysis provides valuable information for heavy vehicle suspension tuning and design. The effects of coupling between the front and rear suspensions on the resulting pitch properties are further investigated using a generalized pitch plane model of a two-axle heavy vehicle. Three performance measures are proposed to describe suspension properties in a dimensionless manner, namely the pitch margin (PM), pitch stiffness ratio (PSR) and coupled pitch stiffness ratio (CPSR). These measures are defined and analyzed for different suspension tuning and load conditions. The effects of suspension tuning on the dynamic responses of the vehicle are evaluated under braking inputs, and excitations arising from different road roughness conditions and driving speeds. Some basic suspension tuning rules of heavy vehicles with unconnected suspensions are proposed on the basis of analytical and simulation results.

2. PITCH PLANE MODELING AND FORMULATIONS

A simplified linear pitch plane model of a two-axle heavy vehicle, involving either unconnected or coupled suspension, is formulated in order to investigate the effect of suspension tuning on the dynamic responses of heavy vehicles. Figure 1 presents the pitch plane vehicle model involving unconnected or coupled suspension. The vehicle model with unconnected suspension, shown in Fig. 1(a), has been widely used in a number of studies for analysis of vehicle ride (Crolla and King, 1999; Odhams and Cebon, 2006). The vehicle model with coupled suspension has also been proposed in a recent study (Odhams and Cebon, 2006), where the front and rear unsprung masses are assumed to be coupled through a massless beam. A generalized pitch-plane vehicle model was formulated comprising an equivalent bump stiffness (k_b) and damping (c_b) with coordinates a_k and a_c, respectively, as shown in Fig. 1(b). The effective pitch stiffness and damping due to the coupled suspension are represented by rotational spring k_p and damping constant c_p. Each tire is represented by a linear spring assuming point-contact with the road and negligible damping.

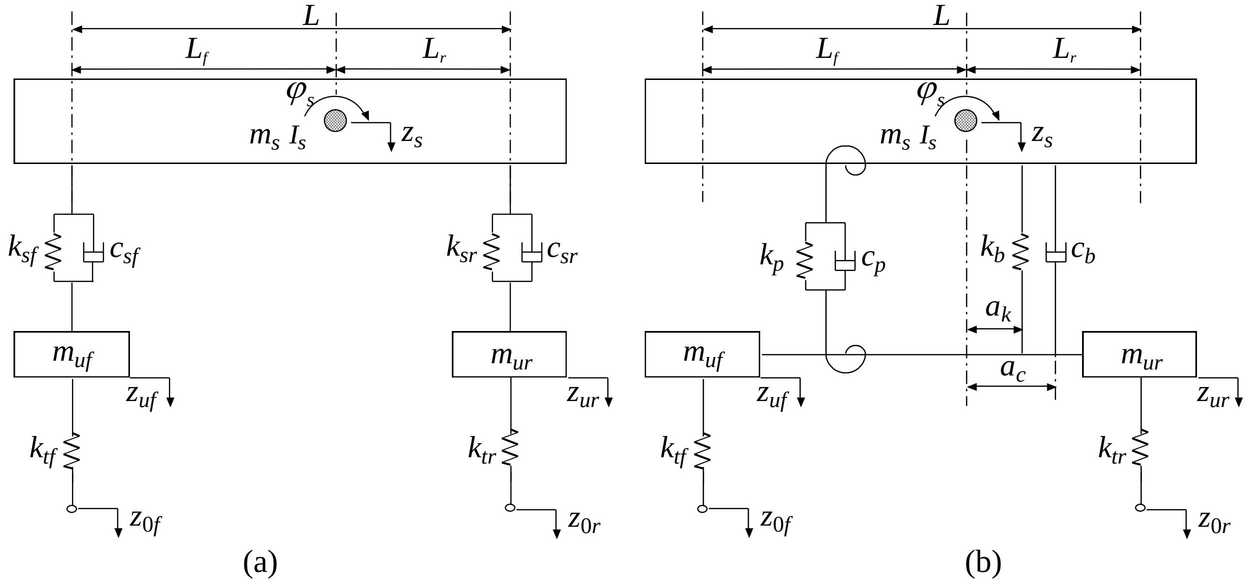


Fig. 1: Pitch plane model of a two-axle heavy vehicle: (a) unconnected suspension; and (b) coupled suspension.

Assuming linear suspension properties, the equations of motion for the model with uncoupled suspension are derived as:

$$m_s\ddot{z}_s - c_{sf}\left(\dot{z}_{uf} - \dot{z}_s + L_f\dot{\varphi}_s\right) - c_{sr}\left(\dot{z}_{ur} - \dot{z}_s - L_r\dot{\varphi}_s\right) - k_{sf}\left(z_{uf} - z_s + L_f\varphi_s\right) - k_{sr}\left(z_{ur} - z_s - L_r\varphi_s\right) = 0$$

$$I_s\ddot{\varphi}_s + c_{sf}L_f\left(\dot{z}_{uf} - \dot{z}_s + L_f\dot{\varphi}_s\right) - c_{sr}L_r\left(\dot{z}_{ur} - \dot{z}_s - L_r\dot{\varphi}_s\right) + k_{sf}L_f\left(z_{uf} - z_s + L_f\varphi_s\right) - k_{sr}L_r\left(z_{ur} - z_s - L_r\varphi_s\right) = 0$$

$$m_{uf}\ddot{z}_{uf} + c_{sf}\left(\dot{z}_{uf} - \dot{z}_s + L_f\dot{\varphi}_s\right) + k_{tf}z_{uf} + k_{sf}\left(z_{uf} - z_s + L_f\varphi_s\right) = k_{tf}z_{0f}$$

$$m_{ur}\ddot{z}_{ur} + c_{sr}\left(\dot{z}_{ur} - \dot{z}_s - L_r\dot{\varphi}_s\right) + k_{tr}z_{ur} + k_{sr}\left(z_{ur} - z_s - L_r\varphi_s\right) = k_{tr}z_{0r} \tag{1}$$

where m_s and I_s are mass and pitch mass moment of inertia of the sprung mass, m_{uf} and m_{ur} are the front and rear unsprung masses, respectively. z_{uf} and z_{ur} are vertical motions of the front and rear unsprung masses, respectively, and z_s and φ_s are vertical and pitch motions of the sprung mass, respectively. L_f and L_r define the longitudinal distances between the sprung mass c.g. and the front and rear axles, respectively, and L is the wheelbase. k_{ti} is vertical stiffness of tires on axle i (i=f,r), and k_{si} and c_{si} (i=f,r) are the vertical stiffness and damping coefficients of suspension at axle i, respectively. z_{0f} and z_{0r} represent the road inputs in the vicinity of the front and rear tire-road contacts, respectively.

The equations of motion for the vehicle model with coupled suspension are derived in a similar manner and expressed as:

$$
\begin{gathered}
m_s \ddot{z}_s - F_{cb} - F_{kb} = 0 \\
I_s \ddot{\varphi}_s + T_{cp} - F_{cb} a_c + T_{kp} - F_{kb} a_k = 0 \\
m_{uf} \ddot{z}_{uf} + \frac{(L_r - a_c)}{L} F_{cb} + \frac{1}{L}(T_{cp} + T_{kp}) + \frac{(L_r - a_k)}{L} F_{kb} = k_{tf}(z_{0f} - z_{uf}) \\
m_{ur} \ddot{z}_{ur} + \frac{(L_f + a_c)}{L} F_{cb} - \frac{1}{L}(T_{cp} + T_{kp}) + \frac{(L_f + a_k)}{L} F_{kb} = k_{tr}(z_{0r} - z_{ur})
\end{gathered}
\tag{2}
$$

where $F_{cb} = c_b\left[\dot{z}_{uf} + (\dot{z}_{ur} - \dot{z}_{uf})\frac{(L_f + a_c)}{L} - \dot{z}_s - a_c \dot{\varphi}_s\right]$, $F_{kb} = k_b\left[z_{uf} + (z_{ur} - z_{uf})\frac{(L_f + a_k)}{L} - z_s - a_k \varphi_s\right]$, $T_{cp} = c_p\left[\dot{\varphi}_s - \frac{(\dot{z}_{ur} - \dot{z}_{uf})}{L}\right]$ and $T_{kp} = k_p\left[\varphi_s - \frac{(z_{ur} - z_{uf})}{L}\right]$. The equations of motions of the two models can be analyzed to develop relationships between the parameters of coupled and unconnected suspensions. The stiffness matrices for the unconnected (K_U) and coupled (K_C) suspension models can be expressed as:

$$
K_U = \begin{bmatrix}
-k_{sf} - k_{sr} & k_{sf} L_f - k_{sr} L_r & k_{sf} & k_{sr} \\
 & -k_{sf} L_f^2 - k_{sr} L_r^2 & -k_{sf} L_f & k_{sr} L_r \\
 & & -k_{sf} - k_{tf} & 0 \\
symmetric & & & -k_{sr} - k_{tr}
\end{bmatrix}
\tag{3}
$$

$$
K_C = \begin{bmatrix}
-k_b & -k_b a_k & k_b \frac{L_r - a_k}{L} & k_b \frac{L_f + a_k}{L} \\
 & -k_b a_k^2 - k_p & -\frac{k_p}{L} + k_b a_k \frac{L_r - a_k}{L} & \frac{k_p}{L} + k_b a_k \frac{L_f + a_k}{L} \\
 & & -\frac{k_p}{L^2} - k_{tf} - k_b \frac{(L_r - a_k)^2}{L^2} & \frac{k_p}{L^2} - k_b \frac{(L_r - a_k)(L_f + a_k)}{L^2} \\
symmetric & & & -\frac{k_p}{L^2} - k_{tr} - k_b \frac{(L_f + a_k)^2}{L^2}
\end{bmatrix}
\tag{4}
$$

Odhams and Cebon (2006) established a number of important relations for the two suspension models by comparing the stiffness matrices in Eqs. (3) and (4). From Eq. (4), it is apparent that $a_k = 0$ yields uncoupled bounce and pitch modes of the sprung mass. The position coordinate a_k is thus termed as the stiffness coupling factor between the bounce and pitch modes. Furthermore, both stiffness matrices are identical under uncoupled bounce and pitch modes, when

$$k_{sf} L_f - k_{sr} L_r = 0; \quad k_b = k_{sf} + k_{sr}; \quad k_p = k_{sf} L_f^2 + k_{sr} L_r^2; \text{ and } \frac{k_p}{k_b} = L_f L_r \tag{5}$$

The matrices K_U and K_C are generally identical under all conditions, when

$$k_b = k_{sf} + k_{sr}; \quad \frac{1}{k_p} = \frac{1}{k_{sf} L^2} + \frac{1}{k_{sr} L^2}; \quad \frac{k_p}{k_b} = (L_f + a_k)(L_r - a_k); \text{ and } \quad a_k = \frac{k_{sr} L_r - k_{sf} L_f}{k_{sf} + k_{sr}} \tag{6}$$

The above suggests that the vehicle model with unconnected suspension can be considered as a special case of the model with coupled suspension. Moreover, it was shown that the relation $a_k = \frac{k_{sr} L_r - k_{sf} L_f}{k_{sf} + k_{sr}}$ is similar to the static margin (SM) defined in vehicle handling analysis, which may be expressed as $SM = \frac{S}{L}$, where $S = \frac{c_{\alpha r} L_r - c_{\alpha f} L_f}{c_{\alpha f} + c_{\alpha r}}$, $c_{\alpha f}$ and $c_{\alpha r}$ are cornering stiffnesses of the front and rear tires, respectively.

For the above relations, it can be further deduced that pitch stiffness of the coupled suspension k_p is equal to two equivalent rotational springs ($k_{sf} L_f^2$ and $k_{sr} L_r^2$) connected in parallel, when bounce and pitch modes are uncoupled. In all the other situations, k_p is equal to the two rotational springs in series. In all conditions, the bump stiffness k_b is invariably equal to the sum of the front and rear spring rates ($k_{sf} + k_{sr}$).

2.1 DIMENSIONLESS MEASURES OF SUSPENSION PROPERTIES

The above relations and discussions can provide a basic understanding of vehicle suspension properties in pitch plane, including uncoupled and coupled. In order to more clearly understand and investigate relationships between suspension tunings of unconnected suspensions and dynamic responses of the heavy vehicles, three dimensionless measures of suspension properties are defined and analyzed in this study, namely pitch margin (PM), pitch stiffness ratio (PSR) and coupled pitch stiffness ratio (CPSR).

PITCH MARGIN (PM)

From the above analysis, it is shown that the bounce spring stiffness is equal to the sum of front and rear suspension spring rates ($k_b = k_{sf} + k_{sr}$), when the two stiffness matrices K_U and K_C are identical. The relations $k_{sf} = k_b \sin^2\theta$ and $k_{sr} = k_b \cos^2\theta$ can therefore be obtained by introducing a variable θ, based on which $a_k = \dfrac{k_{sr}L_r - k_{sf}L_f}{k_{sf} + k_{sr}}$ in Eq. (6) can be rewritten as:

$$a_k = L_r \cos^2\theta - L_f \sin^2\theta \tag{7}$$

By setting $L_f = L\cos^2\gamma$ and $L_r = L\sin^2\gamma$, the pitch margin (PM) can be defined from Eq. (7):

$$PM = \frac{a_k}{L} = \frac{\cos 2\theta - \cos 2\gamma}{2} \tag{8}$$

where $0 < \theta < \pi/2$ and $0 < \gamma < \pi/2$, and γ is a constant for a given load distribution. Equation

(8) would yield a positive value of pitch margin (PM) for $\theta < \gamma$. The pitch margin (PM) of a vehicle suspension can also be described from the dimensionless pitch margin diagram, shown in Fig. 2, where the positive direction of PM is considered to be pointing right. In this diagram, the upper semi-circle of diameter equal to unity scribing ΔACB represents the load distribution, while the lower-semi-circle scribing ΔADB describes the suspension spring rate tuning.

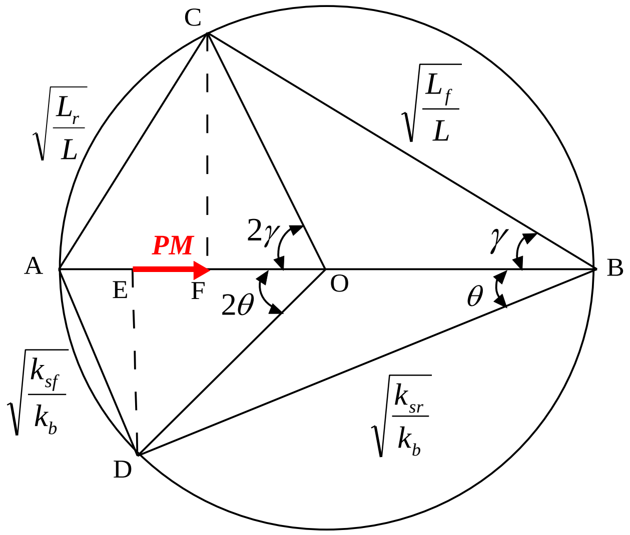


Fig. 2: Dimensionless pitch margin diagram.

For a particular load distribution, Equation (8) can be simplified as:

$$PM = \frac{\cos 2\theta - D}{2} \tag{9}$$

where $D = \cos 2\gamma$ is a constant. The above equation suggests that the PM of a vehicle with a particular load distribution is only a function of θ alone, which itself is related to front/rear suspension stiffness ratio. From the dimensionless pitch margin diagram, it is evident that $PM = \frac{(OE - OF)}{2R} = EF$, where R is the radius. Considering $0 < \theta < \pi/2$, the bounds of PM can be derived as:

$$-\frac{1+D}{2} < PM < \frac{1-D}{2} \tag{10}$$

The above formulation indicates the range of PM corresponding to a given load distribution, which can be varied through tuning of front and rear suspension spring rates.

PITCH STIFFNESS RATIO (PSR)

The pitch stiffness ratio (PSR) is defined as the maximum ratio of pitch to bump rate (k_p / k_b) of a suspension system in a dimensionless manner. From Eq. (6), it can be shown that peak k_p / k_b ratio is obtained for $a_k = \frac{L_r - L_f}{2}$, which yields:

$$PSR = \frac{k_p}{k_b (L/2)^2} \tag{11}$$

Using $k_{sf} = k_b \sin^2 \theta$ and $k_{sr} = k_b \cos^2 \theta$, and the relationship between k_p and k_{sf} and k_{sr} in Eq. (6), the PSR can be obtained as:

$$PSR = \frac{k_b L^2 \sin^2 \theta \cos^2 \theta}{k_b (L/2)^2} = \sin^2 2\theta \tag{12}$$

The above formulation indicates that pitch stiffness ratio (PSR) is only related to the front/rear suspension stiffness ratio or θ, irrespective of load distribution. A relation between PM and PSR can also be derived from Eqs. (9) and (12) as:

$$PSR = 1 - (2PM + D)^2 \tag{13}$$

The above equation suggests that for a vehicle involving unconnected suspension with a particular load distribution, a definite relation exists between the pitch margin (PM) and pitch stiffness ratio (PSR). Equations (12) and (13) further show that for an unconnected suspension, $0 < PSR \leq 1$ and $0 < k_p \leq 0.25 k_b L^2$.

COUPLED PITCH STIFFNESS RATIO (CPSR)

Alternatively, coupled pitch stiffness ratio (CPSR) of an unconnected suspension is also defined upon consideration of the coupling factor a_k between the pitch and bounce modes, such that:

$$CPSR = \frac{k_p + k_b a_k^2}{k_b (L/2)^2} \tag{14}$$

The above can be simplified using Eqs. (6) and (7), such that:

$$CPSR = 1 - 2\cos 2\theta \cos 2\gamma + \cos^2 2\gamma \tag{15}$$

The above relation suggests that unlike PSR, CPSR is also dependent upon the load distribution. Eqs. (8), (12) and (15) further yield a relation among PM, PSR and CPSR:

$$CPSR = PSR + 4PM^2 \tag{16}$$

The above indicates that for $PM = 0$, which implies $a_k = 0$, the pitch and bounce modes are decoupled ($PSR = CPSR$), as observed from the two stiffness matrices K_U and K_C.

For a vehicle with a particular load distribution, γ is a constant, Equation (15) reduces to:

$$CPSR = 1 - 2D\cos 2\theta + D^2 \tag{17}$$

where $D = \cos 2\gamma$. The CPSR reduces to a function of the front/rear suspension stiffness ratio, as in the case of PSR. Considering the limits of suspension stiffness ratio $0 < \theta < \pi/2$, and those of load distribution $0 < \gamma < \pi/2$, Equation (17) can be solved to yield following limiting values of CPSR for different load distributions.

$$\begin{cases} (1-D)^2 < CPSR < (1+D)^2 & D \geq 0 \\ (1+D)^2 < CPSR < (1-D)^2 & D < 0 \end{cases} \tag{18}$$

Equations (9) and (17) further yield a relationship between PM and CPSR for an unconnected suspension, under a particular load condition:

$$CPSR = 1 - 4D * PM - D^2 \tag{19}$$

The above equation indicates that for $D > 0$, when a relatively lower load is supported by the front axle, an increase in the pitch margin (PM) would yield lower CPSR, and vice versa. For $D = 0$, $CPSR = 1$.

2.2 ANALYSIS OF THE THREE MEASURES OF SUSPENSION PROPERTIES

From Eqs. (8), (12) and (15), it is evident that PM and CPSR are functions of both γ and θ, while PSR is a function of θ alone, where $\gamma = \arctan\sqrt{L_r/L_f}$, and $\theta = \arctan\sqrt{k_{sf}/k_{sr}}$. Figures 3(a) and (c) illustrate variations in PM and CPSR as functions of γ and θ, while variations in PSR are presented in Fig. 3(b), as a function of θ. The variations in these measures are presented over a wide range of γ and θ ($0 < \gamma < \pi/2$ and $0 < \theta < \pi/2$). The results indicate that an increase in ratio L_r/L_f increases the pitch margin (PM), while an increase in ratio k_{sf}/k_{sr} causes a decrease in the PM, for a particular load distribution. The PSR achieves a maximum value of 1, for $k_{sf}/k_{sr} = 1$, where increasing or decreasing the ratio k_{sf}/k_{sr} would decrease the PSR, irrespective of the load distribution condition. For the coupled pitch stiffness ratio (CPSR), an increase in the ratio k_{sf}/k_{sr} yields an increase in CPSR for $L_r/L_f < 1$ ($0 < \gamma < \pi/4$). The CPSR assumes a constant unity value, irrespective of the ratio k_{sf}/k_{sr}, when $L_f = L_r$ ($\gamma = \pi/4$). The condition of $L_r/L_f > 1$ ($\pi/4 < \gamma < \pi/2$) yields an opposite trend in CPSR, where an increase in ratio k_{sf}/k_{sr} causes a decrease in the CPSR.

The results suggest that the load distribution and front/rear suspension stiffness ratio strongly affect the proposed dimensionless measures of suspension properties. The load condition and front/rear suspension tuning are also known to significantly influence vehicle dynamic responses. Some relations may therefore exist between the proposed dimensionless measures of suspension pitch properties and vehicle performance characteristics. The following sections will explore the performance characteristics of heavy vehicles under random road excitations as well as braking inputs, for different load conditions and suspension stiffness tunings, in an attempt to establish such relations.

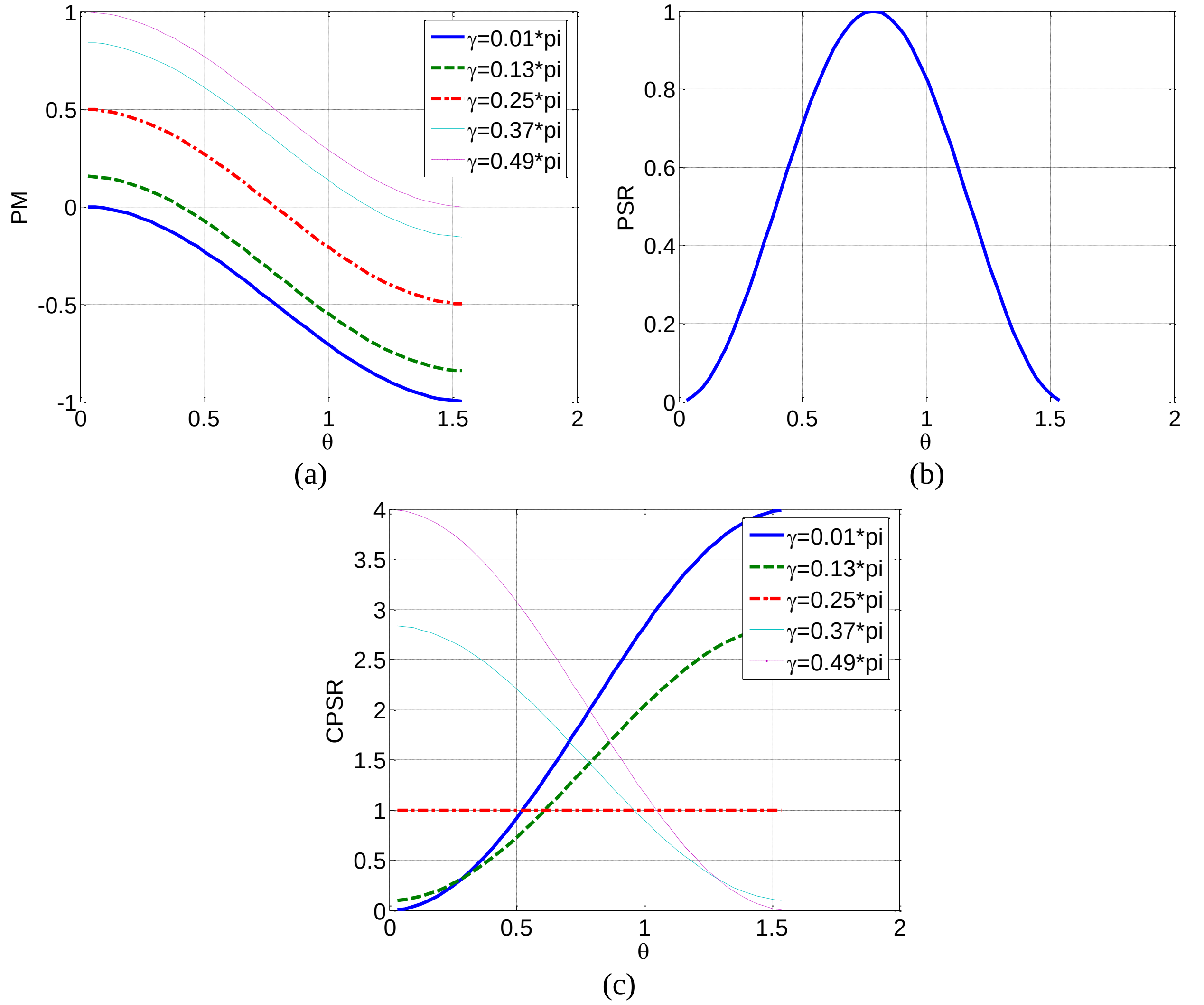


Fig. 3: Variations in measures of suspension properties as functions of γ and θ: (a) PM; (b) PSR; and (c) CPSR.

3. VERTICAL AND PITCH RIDE UNDER RANDOM ROAD EXCITATIONS

Random road roughness is known to be the major excitation for evaluating vehicle ride performance characteristics of alternative suspension designs, which are generally measured in terms of acceleration responses of the vehicles (Gillespie, 1985; ISO 2631-1, 1997; Cole, 2001). The vertical and pitch ride performance of heavy vehicles with different suspension configurations can be conveniently assessed using the pitch plane ride model of road vehicles (Fig. 1), subject to excitations arising from random road elevations. The ride performance is evaluated in terms of: (a) rms vertical acceleration of the sprung mass, measured at five different positions that are evenly distributed along the wheelbase, including the front and rear suspension mountings; and (b) rms pitch acceleration of the sprung mass. The former measure accounts for coupling between the vertical and pitch modes of vibration. The ride analysis are performed under different road roughness inputs, which have been designated as ‘smooth’, ‘medium’ and ‘rough’ on the basis of their relative spatial power spectral densities (PSD) of vertical displacement (Rakheja et al., 2001; Cao et al., 2006a). Figure 4 illustrates the displacement PSD of the three road profiles considered in the study. A relatively wide range of vehicle speeds are also chosen for the analysis (30, 50, 70, 90 and 110 km/h), considering the wheelbase filtering.

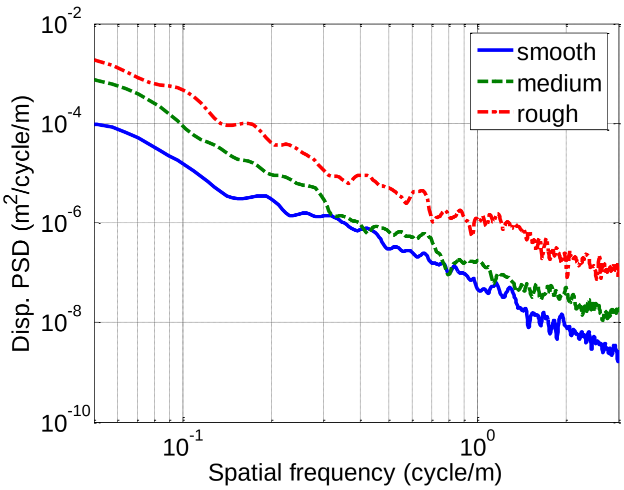
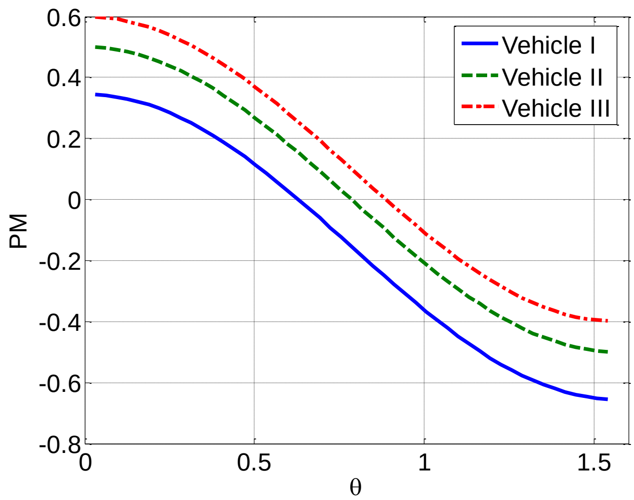


Fig. 4: Displacement PSD of the three road profiles.

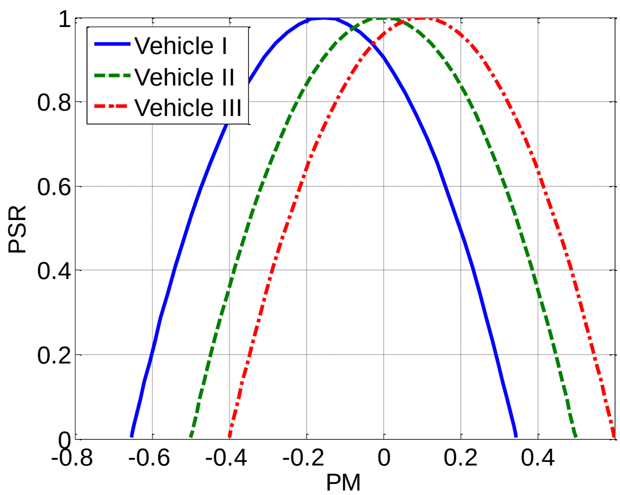
Three heavy vehicle configurations that represent different load distributions are considered for the analysis. The inertial and bounce stiffness parameters are chosen to yield identical bounce mode frequency in the order of 1.2 Hz. The selected model parameters for the three configurations are: (I) $m_s = 15753$ kg, $I_s = 175034$ kgm^2, $k_b = 900$ kN/m, and $L_f / L_r = 1.88$; (II) $m_s = 10932.6$ kg, $I_s = 116689$ kgm^2, $k_b = 624.6$ kN/m, and $L_f / L_r = 1$; and (III) $m_s = 10932.6$ kg, $I_s = 116689$ kgm^2, $k_b = 624.6$ kN/m, and $L_f / L_r = 0.67$. The chosen load distributions yield $D_I = 0.306$, $D_{II} = 0$ and $D_{III} = -0.2$, respectively. The analyses are performed to derive the suspension property measures and dynamic responses of the selected vehicle configurations.

VEHICLE CONFIGURATION I

The heavy vehicle configuration I involves greater load distributed on the rear axle, which yields $\gamma = 0.63$ rad. The variations in PM of the suspension are evaluated as a function of θ, ranging from 0 to $\pi/2$, and shown in Fig. 5(a). The pitch stiffness ratio (PSR) and coupled pitch stiffness ratio (CPSR) are evaluated as a function of the pitch margin (PM), as illustrated in Figs. 5(b) and (c), respectively. The results show that an increase in θ (or ratio k_{sf}/k_{sr}) reduces the PM, while the CPSR decreases with an increase in PM in a linear manner. The PSR increases with increasing PM, and achieves the maximum value of 1, when $PM = -D/2$. A further increase in PM yields a reduction in PSR, as evident in Fig. 5(b).



(a)



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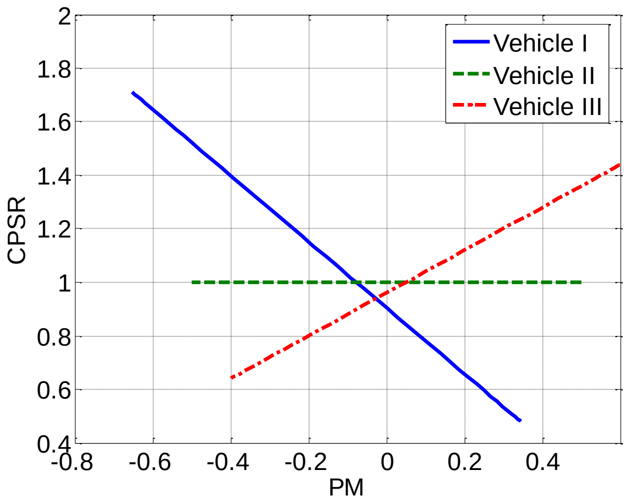


(c)

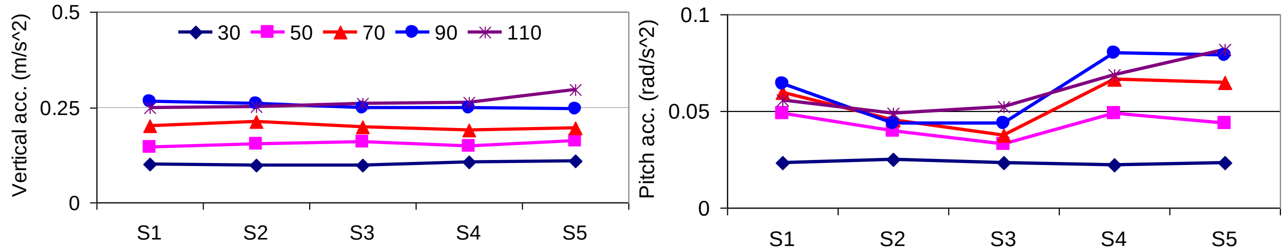
Fig. 5: Variations in suspension property measures of different vehicle configurations: (a) PM vs θ; (b) PSR as a function of PM; and (c) CPSR as a function of PM.

The above results show that the proposed pitch measures are strongly dependent upon the front to rear suspension spring rate ratio. The influence of front and rear suspension rates on the resulting properties are evaluated for five different stiffness tunings of the linear unconnected suspension, which are summarized in Table 1. The sum of the front and rear suspension spring rates is held constant ($k_b = 900$ kN/m) for all the five stiffness tunings considered, so as to maintain the total bounce stiffness of the vehicle unaffected (Crolla and King, 1999). For the selected distributions, tuning S1 results in nearly identical natural frequencies for both the front and rear suspensions, while S3 and S5 cause lowest and highest front suspension natural frequencies, respectively. The tuning S1 provides identical values of PSR and CPSR, while S3 and S5 cause lowest and highest values of PSR and CPSR, respectively, as illustrated in Table 1. The tuning S1 that yields $PM = 0$ suggests decoupled front and rear suspensions, often considered as theoretically ideal suspension tuning for ride comfort (Wong, 2001). A relatively lower value of front suspension rate (S2 and S3) yields $PM > 0$, while higher front suspension rate (S4 and S5) results in $PM < 0$.

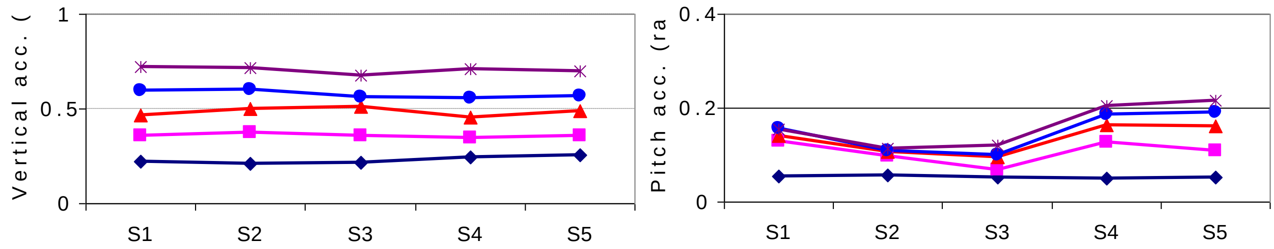
Table 1: Influence of front and rear suspension spring rate ratio on the suspension property measures of vehicle configuration I.

Tuning	k_{sf} (kN/m)	k_{sr} (kN/m)	θ (rad)	PM	PSR	CPSR
S1	312.3	587.7	0.63	0	0.906	0.906
S2	222.3	677.7	0.52	0.1	0.744	0.784
S3	132.3	767.7	0.39	0.2	0.502	0.661
S4	402.3	497.7	0.73	-0.1	0.989	1.029
S5	492.3	407.7	0.83	-0.2	0.991	1.151

The vertical and pitch acceleration responses to random road excitations are evaluated for different suspension tunings and vehicle speeds. The damping ratios of the front and rear suspensions are tuned to achieve a value of 0.2 for all the suspension tunings considered in the analysis. The vertical responses assessed at five selected locations, and pitch responses are expressed in terms of rms accelerations. The vertical ride is then expressed as the root mean value of the rms accelerations evaluated at the five positions ($\sqrt{\sum_{n=1}^{5} \ddot{z}_{rmsn}^2 / 5}$). Figure 6 presents comparisons of rms vertical and pitch acceleration responses of different tunings, under the three random road inputs and a range of vehicle speeds. The results generally show comparable vertical responses of different suspension tunings under given road roughness and vehicle speed conditions, although slight variations are also evident, which may be partly attributed to wheelbase filtering effect. The rms vertical acceleration generally tends to increase with increasing speed and road roughness. Unlike the vertical acceleration response, the pitch acceleration response is strongly influenced by the suspension stiffness tunings, except at the low speed of 30 km/h. The suspension tunings S2 and S3 with positive PM and relatively lower PSR and CPSR yield considerably lower pitch acceleration, irrespective of the road roughness and vehicle speed above 30 km/h, as seen in Fig. 6. The S4 and S5 tunings with negative PM and higher PSR and CPSR, on the other hand, result in considerably larger pitch acceleration responses, compared to S2 and S3 at the speeds above 30 km/h.



(a)



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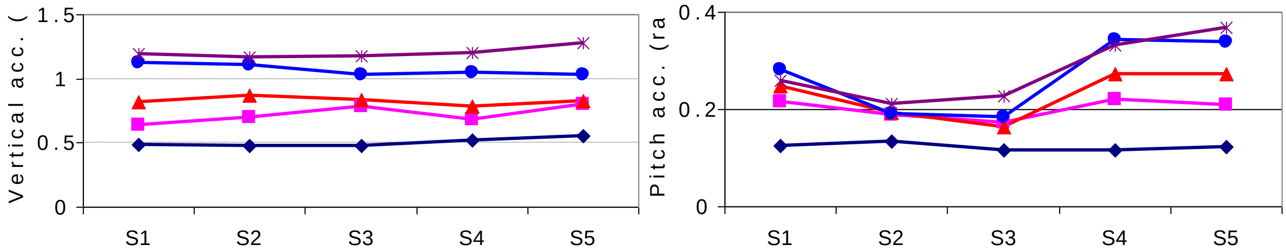


(c)

Fig. 6: Comparisons of rms vertical and pitch acceleration responses of vehicle configuration I with

different suspension tunings under random road inputs: (a) smooth; (b) medium; and (c) rough.

VEHICLE CONFIGURATION II

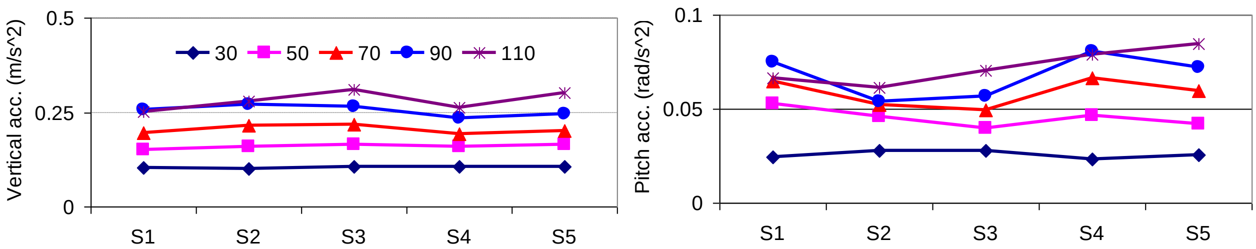
Unlike the vehicle configuration I, the vehicle configuration II exhibits an identical load distribution on each axle, which yields $\gamma = \pi/4$ rad. The variations in PM with θ ($0 < \theta < \pi/2$) is presented in Fig. 5(a). The corresponding variations in PSR and CPSR as a function of PM are illustrated in Figs. 5(b) and (c), respectively. The results indicate that the PM decreases with an increase in θ, as observed for vehicle configuration I. The CPSR maintains a constant value of 1, irrespective of the PM. The PSR, however, achieves the maximum value of 1 for *PM*=0, and decreases with increasing PM in both directions in a symmetric manner, as evident in Fig. 5(b).

Table 2 summarizes the influence of various suspension stiffness tunings (S1 to S5) of the linear unconnected suspension on the suspension property measures of the vehicle configuration. In this case, the front and rear suspension rates are varied to attain total suspension spring rate ($k_b = k_{sf} + k_{sr}$) of 624.6 kN/m for all the five tunings. For the even load condition, tuning S1 yields identical natural frequencies of the front and rear suspensions, while S3 and S5 tunings would yield lowest and highest front suspension natural frequencies, respectively. The tuning S1 yields a PM of 0, indicating decoupled bounce and pitch vibration modes. The tunings S2 and S3 with relatively lower front suspension rates have positive PMs of 0.144 and 0.288, respectively. The S4 and S5 tunings with relatively higher front suspension rates, on the other hand, yield negative PM. Owing to the symmetric variations in PSR, the tunings S2 and S4 yield identical values of PSR, which are lower than that of S1 tuning, while tunings S3 and S5 yield lowest PSR. Unlike the PSR, the value of CPSR remains 1 for all the tunings.

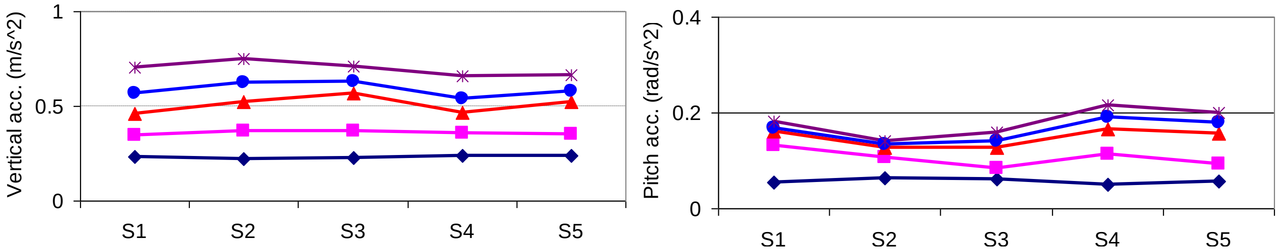
Table 2: Influence of front and rear suspension spring rate ratio on the suspension property measures of vehicle configuration II.

Tuning	k_{sf} (kN/m)	k_{sr} (kN/m)	θ (rad)	PM	PSR	CPSR
S1	312.3	312.3	0.79	0	1	1
S2	222.3	402.3	0.64	0.144	0.917	1
S3	132.3	492.3	0.48	0.288	0.668	1
S4	402.3	222.3	0.93	-0.144	0.917	1
S5	492.3	132.3	1.09	-0.288	0.668	1

The relative ride responses of the vehicle configuration II with different suspension tunings are also evaluated under the three random road excitations and different vehicle speeds. The damping ratios of the front and rear suspension are tuned to realize identical value of 0.2 for all the suspension tunings. Figure 7 presents comparisons of rms vertical and pitch acceleration responses of the vehicle involving different suspension tunings and operating conditions (speed and road roughness). Unlike the configuration I, the suspension tunings seem to have a strong effect on the vertical rms acceleration response of the vehicle configuration II, particularly at higher speeds and rougher road. Reducing the front suspension spring rate (S2 and S3) yields higher vertical acceleration responses. The rms pitch acceleration responses of the sprung mass are significantly different due to different tunings, as observed for configuration I. Compared to S1 tuning, the tunings S2 and S3 with positive PM and relatively lower PSR and k_{sf}, could significantly reduce pitch acceleration responses of the vehicle at speeds above 30 km/h, irrespective of the road roughness. These two tunings, however, tend to slightly increase the pitch acceleration responses at the low speed of 30 km/h, which has also been observed in the reported studies (Sharp and Pilbeam, 1993; Sharp, 2002). Tunings S4 and S5 with negative PM, relatively lower PSR and higher k_{sf}, however, exhibit considerably higher pitch acceleration responses, compared to those of S2 and S3 at speeds above 30 km/h.



(a)



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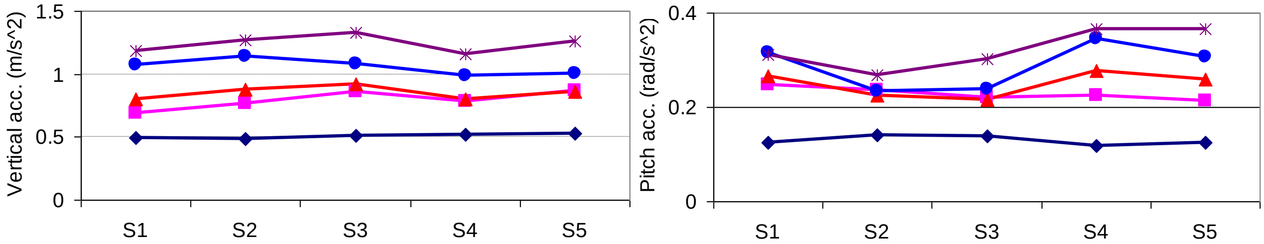


(c)

Fig. 7: Comparisons of rms vertical and pitch acceleration responses of vehicle configuration II with different suspension tunings under random road inputs: (a) smooth; (b) medium; and (c) rough.

VEHICLE CONFIGURATION III

The vehicle configuration III involves relatively larger load on the front axle ($L_f/L_r = 0.67$) with $\gamma = 0.89$ rad. Figure 5(a) presents the variations in PM as a function of θ, while the variations in PSR and CPSR are illustrated in Figs. 5(b) and (c), as a function of PM. The results show that the PM decreases with increasing θ (or ratio k_{sf}/k_{sr}), as in case of configurations I and II. While the CPSR increases linearly with an increase in PM, the PSR achieves its peak value of 1 for *PM*=0.1, where PSR decreases with increasing PM in both directions.

The dimensionless measures of suspension properties of configuration III are evaluated for the five different spring rate tunings, in a similar manner, where the total spring rate k_b is held as 624.6 kN/m, as summarized in Table 3. Tuning S1 yields nearly identical natural frequencies for both front and rear suspensions, while S3 and S5 tunings would yield lowest and highest front suspension natural frequencies, respectively. Tuning S1 that yields a PM of 0, suggests decoupling between the bounce and pitch vibration modes of the sprung mass, while tunings S2 and S3 yield positive values of PM. The tunings S4 and S5, on the other hand, exhibit negative PM values. The tuning S2 with identical front and rear suspension rates yields higher PSR than S1 and S3 tunings, which show identical PSR. Tunings S4 and S5 with relatively lower rear suspension rates exhibit relatively lower PSR. Unlike the PSR, the CPSR of S3 tuning is highest among the five tunings considered.

Table 3: Influence of front and rear suspension spring rate ratio on the suspension property measures of vehicle configuration III.

Tuning	k_{sf} (kN/m)	k_{sr} (kN/m)	θ (rad)	PM	PSR	CPSR
S1	374.76	249.84	0.89	0	0.96	0.96
S2	312.3	312.3	0.79	0.1	1	1.04
S3	249.84	374.76	0.67	0.2	0.96	1.12
S4	437.22	187.38	0.99	-0.1	0.84	0.88
S5	499.68	124.92	1.11	-0.2	0.64	0.8

The rms vertical and pitch acceleration responses of the vehicle configuration III involving different suspension tunings are also evaluated under different random road inputs and vehicle speeds, while the damping ratios of the front and rear suspension for each tuning are selected as 0.2. Figure 8 illustrates comparisons of rms vertical and pitch acceleration responses of different tunings for different road roughness and speed inputs. The results show that the vehicle with all the five different suspension tunings exhibits similar vertical acceleration responses for the low speed of 30 km/h, as it was observed for configurations I and II. A relatively softer front suspension (S2 and S3), however, yields higher rms vertical acceleration at speeds above 30 km/h. This trend is similar to that observed for configuration II. The pitch acceleration responses of S4 and S5 tunings with negative PM, and relatively higher k_{sf} and lower PSR, are comparable with those of S1, but considerably larger than those of S2 and S3 tunings with positive PM, for vehicle speeds above 30 km/h. For the low speed of 30 km/h, the rms pitch acceleration responses of S2 and S3, however, are higher than those of S1, S4 and S5 tunings.

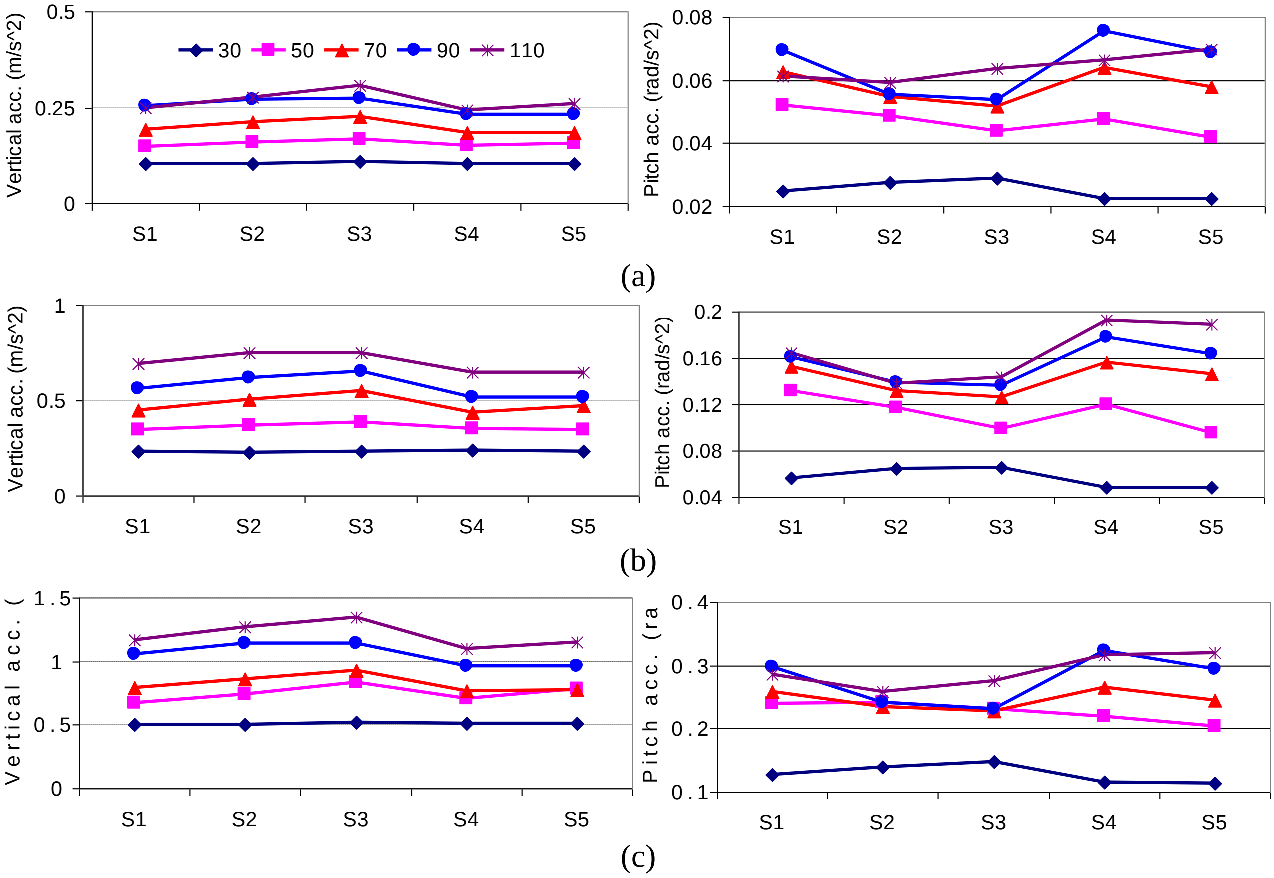


Fig. 8: Comparisons of rms vertical and pitch acceleration responses of vehicle configuration III with different suspension tunings under random road inputs: (a) smooth; (b) medium; and (c) rough.

3.1 DISCUSSIONS

From the above results attained under a wide range of random road roughness inputs and vehicle speeds, it is evident that the suspension stiffness tunings could strongly affect the pitch acceleration responses, while their influence on vertical acceleration responses is strongly dependent upon road roughness characteristics. The above analyses, however, do not reveal clear relationships between the vertical and pitch ride responses and the proposed dimensionless measures (PM, PSR and CPSR). This is mainly due to different wheelbase filtering effects under various speeds and variations in load distributions considered in the study. An alternative simplified quantitative measure, based on the sum of acceleration responses of a particular suspension tuning under different speeds and road roughness inputs normalized with respect to that of tuning S1, is explored in an attempt to establish a more definite relationship. This measure is referred to as normalized cumulative acceleration (NCA), which is expected to reduce the contribution due to wheelbase filtering, while it imposes equal weighting for various road roughness inputs. The effect of load variations may thus be observed more clearly from the proposed alternative measure.

Table 4 summarizes the NCA measures of vertical and pitch acceleration responses of the three vehicle configurations with different suspension tunings (S1 to S5) together with the corresponding values of PM, PSR and CPSR. An NCA value of a particular tuning below 1 implies lower cumulative acceleration response than the baseline tuning S1 that yields identical front and rear suspension frequencies. The NCA>1 would imply larger cumulative acceleration response than that of the S1 tuning. The variations in VCA measures do not show a clear trend with CPSR. From the results, following relations of the vertical and pitch mode NCA with the dimensionless measures of the suspension could be obtained:

- Maintaining a constant value of sum of the front and rear suspension spring rates would still induce variations in vertical acceleration responses, irrespective of the load distribution, which is attributed to coupling between the bounce and pitch modes of the sprung mass.
- A positive value of PM generally yields a higher value of vertical NCA, suggesting deterioration of vertical ride, particularly when $L_f/L_r = 1$ (configuration II) and $L_f/L_r < 1$ (configuration III).
- A positive value of PM, however, generally yields considerably lower values of pitch NCA, irrespective of the load distribution.
- The PM and PSR show coupled effects on both vertical and pitch NCA. A lower value of PSR coupled with a positive PM yields a considerable reduction in pitch NCA.
- For typical heavy vehicles ($L_f/L_r > 1$), a positive PM tuning would yield a lower value of PSR and thus significant improvement in the pitch ride performance. The corresponding change in vertical ride is very small.

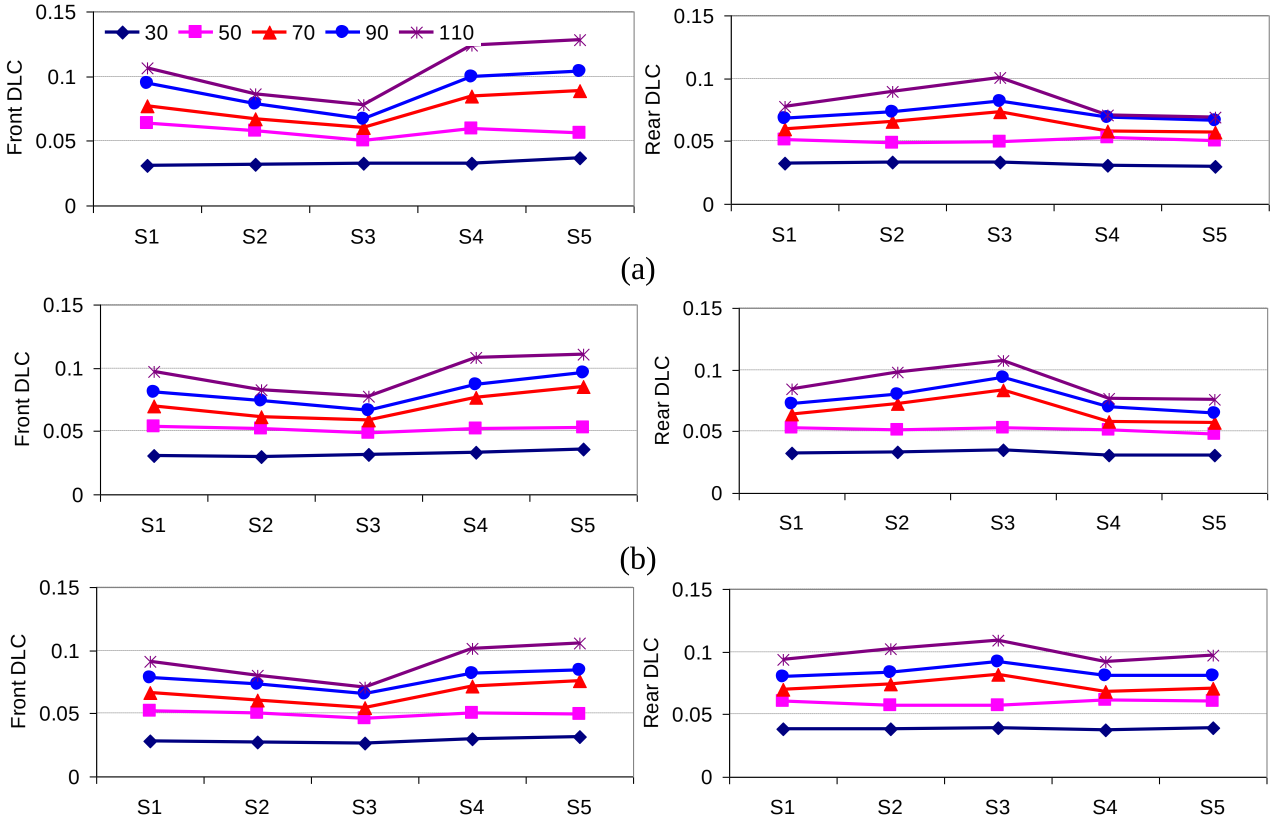
- For vehicles with either even or greater load on the front axle, a positive PM tuning is beneficial for pitch ride performance. An appropriate negative PM tuning, on the other hand, could yield considerable vertical ride improvement with only slight increase in pitch NCA.

Table 4: Comparisons of NCA of vehicle configurations with different suspension tunings.

Tuning	PM	PSR	CPSR	Vertical NCA	Pitch NCA
Configuration I					
S1	0	0.906	0.906	1	1
S2	0.1	0.744	0.784	1.016	0.794
S3	0.2	0.502	0.661	0.983	0.735
S4	-0.1	0.989	1.029	0.989	1.142
S5	-0.2	0.991	1.151	1.048	1.156
Configuration II					
S1	0	1	1	1	1
S2	0.144	0.917	1	1.072	0.846
S3	0.288	0.668	1	1.027	0.853
S4	-0.144	0.917	1	0.901	1.05
S5	-0.288	0.668	1	1.061	0.993
Configuration III					
S1	0	0.96	0.96	1	1
S2	0.1	1	1.04	1.079	0.906
S3	0.2	0.96	1.12	1.053	0.891
S4	-0.1	0.84	0.88	0.849	1.03
S5	-0.2	0.64	0.8	1.024	0.967

4. DYNAMIC TIRE LOAD RESPONSES

Dynamic tire loads of road vehicles, especially heavy vehicles, are known to accelerate road damages. The relative dynamic tire load responses of different vehicle configurations involving different suspension tunings are therefore assessed under different random road inputs and vehicle speeds, as a measure of road-friendliness characteristics of the vehicles. For a given speed, the dynamic tire load responses of the vehicle with a particular tuning revealed similar trends under excitations arising from different road roughness. As an example, Figure 10 illustrates the dynamic load coefficients (DLC) due to front and rear wheels of the three vehicle configurations with different suspension tunings (described in Tables 1 to 3), subject to the excitations from medium road. The DLC due to tire force is evaluated as the ratio of rms dynamic tire force to the static tire force (Cebon, 1999). The front or rear tire force DLC responses exhibit very similar trends, for the three vehicle configurations considered. The results show that the influence of suspension tuning on the DLC is relatively insignificant at the low speed of 30 km/h, irrespective of the load distribution. The suspension tuning, however, strongly influence the DLC responses at higher speeds. The S2 and S3 tunings with relatively lower front suspension rate and thus positive PM yield lower magnitudes of DLC at the front wheel, but higher magnitudes of DLC at the rear. As expected, the tunings S4 and S5 with relatively higher front suspension stiffness exhibit opposite trends, as seen in Fig. 9. The results also suggest that a relatively soft suspension is helpful for improving dynamic tire load performance of a heavy vehicle, which is consistent with the results reported in many studies (Rakheja and Woodrooffe, 1996; Cebon, 1999).



(a)

(b)

(c)

Fig. 9: DLC responses of different vehicle configurations with different suspension tunings under medium road inputs: (a) configuration I; (b) configuration II; and (c) configuration III.

In order to further compare the effects of different suspension tunings on the overall dynamic tire load characteristics of the vehicles, a simplified quantitative measure is formulated on the basis of normalized cumulative DLC (NCD). This measure of a vehicle with a particular suspension tuning is derived upon summing the DLC values due to the front and rear tire forces attained for different road roughness and speed conditions considered in this study. The resulting cumulative DLC is normalized with respect to that of the vehicle with tuning S1.

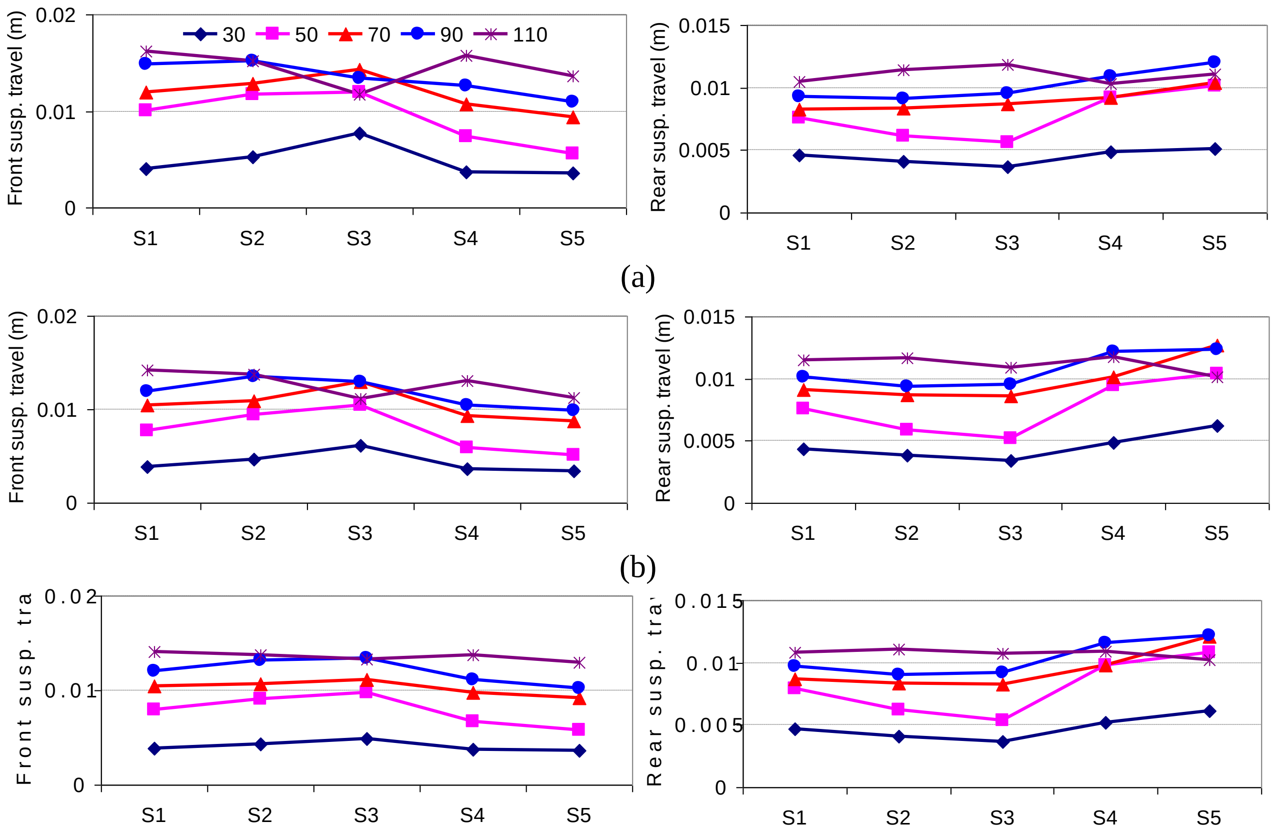
Table 5 summarizes the NCD responses of the three vehicle configurations with five different suspension tunings, together with the corresponding PM values. The results suggest that suspension tuning with a negative value of PM tends to deteriorate the cumulative DLC of vehicle, irrespective of the load distribution and road roughness. A positive tuning, however, could yield slightly lower values of cumulative DLC, except when $L_f / L_r = 1$. The results indicate that positive PM tuning is desirable for improved pitch ride and dynamic tire load performance.

Table 5: Comparisons of normalized cumulative DLC (NCD) responses of vehicle configurations with different suspension tunings.

Tuning	Configuration I		Configuration II		Configuration III	
	PM	NCD	PM	NCD	PM	NCD
S1	0	1	0	1	0	1
S2	0.1	0.972	0.144	0.995	0.1	0.982
S3	0.2	0.992	0.288	1.056	0.2	0.98
S4	-0.1	1.03	-0.144	1.016	-0.1	1.032
S5	-0.2	1.062	-0.288	1.064	-0.2	1.086

5. SUSPENSION TRAVEL RESPONSES

Suspension designs and tunings of road vehicles are subjected to the constraints posed by rattle space. Although a very soft suspension is known to be beneficial for ride comfort and road friendliness of vehicles, it could induce significantly larger suspension travel. The suspension travel responses of the three vehicle configurations with different suspension tunings are assessed under different random road excitations and speeds. The rms travel responses of a suspension with a particular tuning were observed to depend upon the road roughness in a manner similar to that observed for dynamic tire loads. The influences of the suspension tuning on the rms suspension travel, however, are relatively smaller when compared to those on DLC. Figure 10, as an example, presents the front and rear suspension travel responses of the three vehicle configurations involving different suspension tunings under the medium-rough road input at various speeds. Suspension tunings with PM<0 (S4 and S5) yield lower front suspension travel but higher rear suspension travel, which is attributed to relatively stiffer and softer front and rear suspensions, respectively. The responses under the higher speed of 100 km/h, however, form an exception, which may be partly due to greater contribution of wheelbase filtering. With the exception of responses at 110 km/h, the tunings S2 and S3 with PM>0 yield only minimal influences on the travel responses. The rms magnitudes tend to be slightly larger at the front and lower at the rear, when compared to those of tuning S1.



(c)

Fig. 10: Suspension travel responses of vehicle configurations with different suspension tunings under medium road inputs: (a) configuration I; (b) configuration II; and (c) configuration III.

The effect of suspension tuning on the overall suspension travel performance of the vehicles is further evaluated, on the basis of a simplified quantitative measure, namely normalized cumulative suspension travel (NCST). For a vehicle configuration with a particular suspension tuning, NCST is formulated as the sum of the front and rear suspension travels for different speeds and road inputs normalized with respect to that of the S1. Table 6 summarizes the NCST responses of the three vehicle configurations with different suspension tunings, as well as the PM values. From the results, it is apparent that influences on the NCST are very small for the range of tunings considered. The results shown in Fig. 10 and Table 6 suggest that a particular tuning may yield slightly lower travel of suspension at one of the axles, while the response at the other axle may be slightly higher. Suspension tuning with PM>0 tends to slightly deteriorate the cumulative suspension travel response of the vehicles, when $L_f / L_r \geq 1$. A negative PM tuning, however, could slightly improve the cumulative suspension travel response, when $L_f / L_r \geq 1$.

Table 6: Comparisons of normalized cumulative suspension travel (NCST) responses of vehicle configurations with different suspension tunings.

Tuning	Configuration I		Configuration II		Configuration III	
	PM	NCST	PM	NCST	PM	NCST
S1	0	1	0	1	0	1
S2	0.1	1.022	0.144	1.007	0.1	0.998
S3	0.2	1.019	0.288	1.006	0.2	0.997
S4	-0.1	0.972	-0.144	0.997	-0.1	1.013
S5	-0.2	0.949	-0.288	0.996	-0.2	1.027

6. PITCH ATTITUDE RESPONSES UNDER BRAKING INPUTS

The variations in vehicle pitch attitude induced by braking maneuvers could induce the variations in normal tire loads and thus affect the handling quality of vehicles (Dahlberg, 1999; Wong, 2001; Kang et al., 2002). The influence of suspension tuning on the vehicle response to braking inputs are further investigated using a pitch and vertical dynamics model of the vehicle. A pitch plane braking model of a heavy vehicle, shown in Fig. 11 (Cao et al., 2007), is used to investigate the braking responses of the three vehicle configuration coupled with different suspension tunings in terms of peak pitch angle and suspension travel. The model is sufficiently generalized for investigating coupled as well as unconnected suspension systems, and it incorporates: longitudinal motion (x) of the vehicle, vertical motions of the front and rear unsprung masses (z_{uf}, z_{ur}), vertical (z_s) and pitch (φ_s) motions of the sprung mass, and angular velocities of the front and rear wheels (ω_f, ω_r). The vertical properties of tires are represented by linear stiffness and damping elements, assuming point-contact with the road surface. Figure 11(b) illustrates the forces and moments considered to act on a wheel and tire assembly under braking. The equations of motion for the pitch plane vehicle model are formulated under excitations arising from the vehicle-road interactions and the braking torque. The formulations include total suspension forces, comprising the static and dynamic forces developed by the front (f_f) and rear (f_r) suspensions. Assuming small pitch motions, the equations of motion are summarized below:

$$
\begin{aligned}
m_s\ddot{z}_s &= -f_f - f_r + m_s g \\
I_s\ddot{\varphi}_s &= f_f L_f - f_r L_r - f_{xf}(h + z_{0f} - z_s) - f_{xr}(h + z_{0r} - z_s) \\
m_{uf}\ddot{z}_{uf} &= f_f + k_{tf}(z_{0f} - z_{uf}) + c_{tf}(\dot{z}_{0f} - \dot{z}_{uf}) + m_{uf} g \\
m_{ur}\ddot{z}_{ur} &= f_r + k_{tr}(z_{0r} - z_{ur}) + c_{tr}(\dot{z}_{0r} - \dot{z}_{ur}) + m_{ur} g \\
(m_s + m_{uf} + m_{ur})\ddot{x} &= -(f_{xf} + f_{xr}) - m_s g \varphi_s \\
I_{wi}\dot{\omega}_i &= f_{xi} \cdot r_i - T_{bi} \quad (i = f, r)
\end{aligned}
\tag{20}
$$

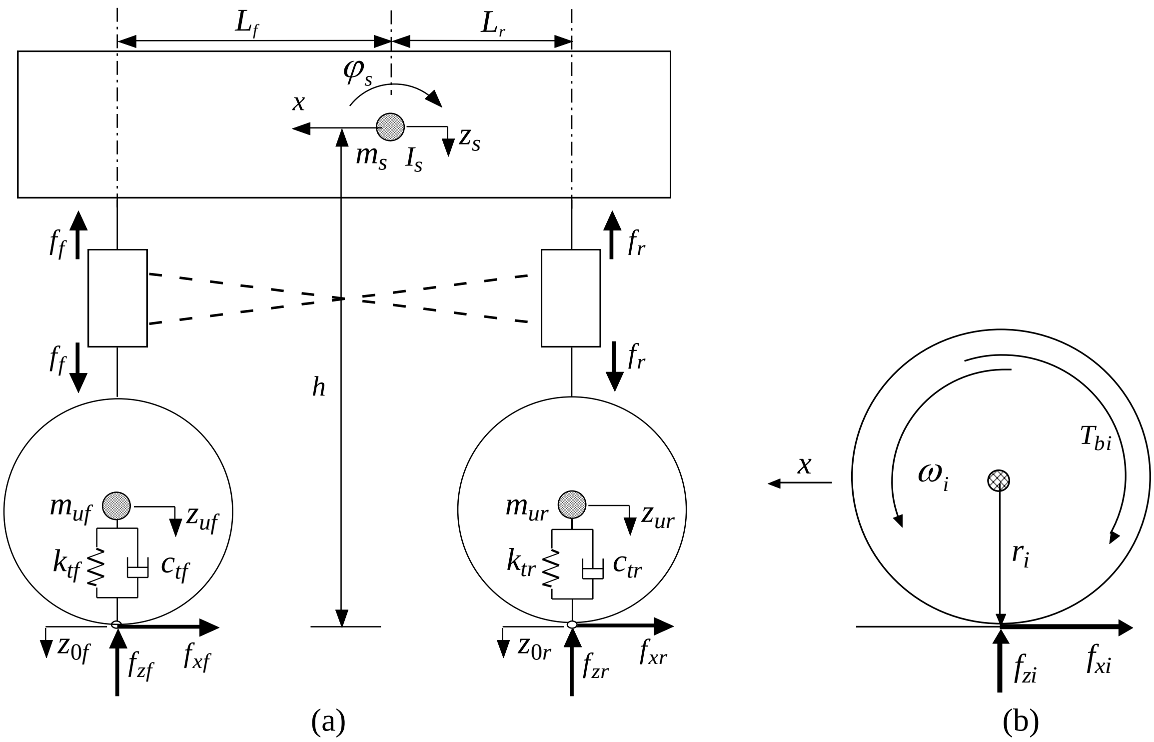


Fig. 11: (a) Pitch plane braking model of a two-axle heavy vehicle; and (b) forces and moments acting on a wheel and tire assembly under braking.

where c_{ti} ($i=f,r$) is vertical damping coefficient of tire, f_{xf} and f_{xr} are braking efforts developed by the front- and rear-axle tires, respectively, and f_{zf} and f_{zr} are the respective normal forces applied to the road surface, h is vehicle c.g. height from the ground, T_{bi} is applied braking torque, r_i is effective radius of tire i, and I_{wi} is polar mass moment of inertia of wheel i. The Magic Formula tire model was utilized to derive the braking forces developed by the tires, as a function of slip and normal load (Bakker et al., 1987; Pacejka and Bakker, 1991). The validation of the vehicle model was investigated in view of the measured data reported by Murphy et al. (1972), under different load conditions and braking inputs. Comparisons of results attained from the model with the reported measured data showed reasonably good agreement between them.

The vehicle model is analyzed for the three configurations and five suspension tunings under a braking input. The total braking gain was chosen as 98.2 Nm/kPa for all the three configurations (Fancher et al., 1986; Cao et al., 2007). The sum of the driver's reaction time and the braking system time lag was set as 0.75 s, while the rise time of the braking system was set as 0.25 s

(Fancher et al., 1986; Delaigue and Eskandarian, 2004). The initial vehicle speed for the analyses was set as 100 km/h, while the braking input was selected as 172 kPa for all the three vehicle configurations considered. Figure 12 presents the peak pitch angle and peak suspension travel responses of the three vehicle configurations with different suspension tunings under the selected braking input. The results show that both the peak pitch angle and suspension travel responses are strongly influenced by suspension tuning, and exhibit similar trends for a particular load distribution. The peak pitch angle and peak suspension travel responses during braking are directly related to the suspension PSR, defined in Eq. (11). An examination of the PSR values of different suspension tunings, presented in Table 4, and the peak responses suggests that an increase in suspension PSR would yield considerably lower peak pitch angle and suspension travel responses under braking inputs.

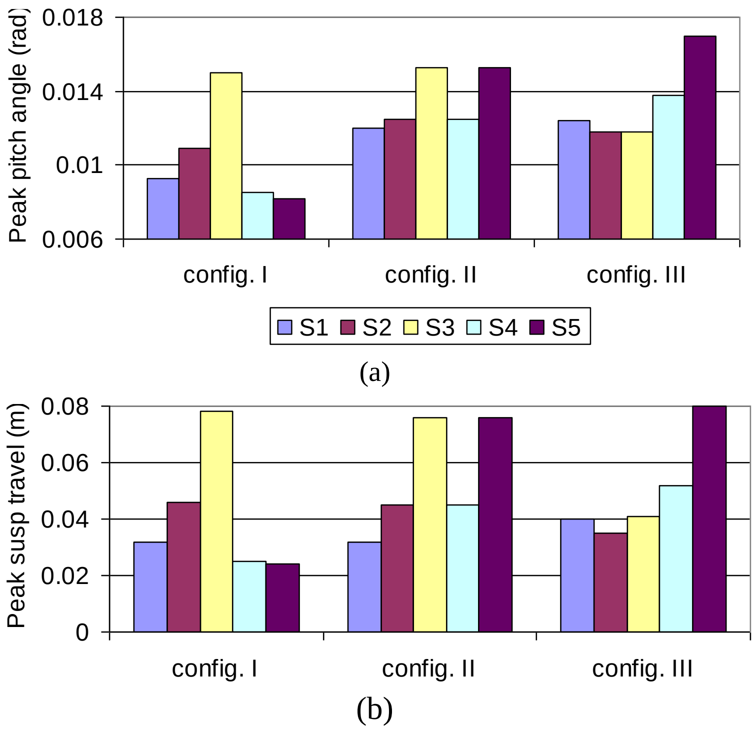


Fig. 12: Dynamic responses of different vehicle configurations with different suspension tunings under braking inputs: (a) peak pitch angle responses; and (b) peak suspension travel responses.

The results attained from Section 3 and Fig. 12 indicate that for typical heavy vehicles with greater load on the rear axle, a positive PM tuning that yields a relatively lower PSR could considerably enhance pitch ride with insignificant influence on vertical ride. Such suspension tuning, however, would significantly deteriorate the pitch attitude control and suspension travel performance during braking maneuvers. This is attributed to the definite relationship between PM and PSR of an unconnected suspension for a given load distribution. Pitch plane coupled suspensions, however, could offer considerable potential for independent tuning of PM and PSR, to achieve improved ride and handling performance of heavy vehicles.

7. CONCLUSIONS

The front/rear suspension stiffness tunings of two-axle heavy vehicles with unconnected suspensions were systematically explored in this study, under a wide range of random road inputs and driving speeds, as well as braking inputs. Upon considerations of the mathematical formulations of two pitch plane models of a two-axle heavy vehicle with unconnected and coupled suspensions, three dimensionless measures of suspension properties, namely the pitch margin (PM), pitch stiffness ratio (PSR) and coupled pitch stiffness ratio (CPSR), were proposed and analyzed for different unconnected suspension tunings and load conditions. The simulation results were explored in an attempt to derive influences of suspension tunings and measures on the responses, and for establishing some basic suspension tuning rules of heavy vehicles with conventional unconnected suspensions. The major findings of the study are summarized below.

- Maintaining a constant value of sum of the front and rear suspension spring rates would still induce variations in vertical acceleration responses, irrespective of the load distribution, which is attributed to the coupling effect between the bounce and pitch modes of the sprung mass.
- For a vehicle with a particular load distribution, the use of an unconnected suspension yields a definite relationship between the dimensionless suspension property measures, namely the pitch margin (PM) and pitch stiffness ratio (PSR).
- A positive value of PM could considerably improve pitch ride, irrespective of the load

distribution. It, however, generally deteriorates vertical ride, particularly for vehicles with even load distribution or greater load on the front axle.

- The PM and PSR show coupled effects on both vertical and pitch ride. A lower value of PSR together with a positive PM value could yield considerable improvement in the pitch ride.
- The peak pitch angle and peak suspension travel responses during braking are directly related to PSR. A higher PSR would be desirable for reducing both the peak pitch angle and peak suspension travel responses during braking maneuvers.
- For typical heavy vehicles with greater load on the rear axle, a positive PM tuning that yields a relatively lower PSR could considerably enhance the pitch ride with only negligible influence on vertical ride response. Such suspension tuning, however, would significantly deteriorate the pitch attitude control and suspension travel responses of the vehicles during braking maneuvers.
- A positive PM tuning also represents a relatively lower front suspension natural frequency, which indicates a lower roll stiffness of the front suspension. The use of anti-roll bar or roll plane coupled suspension would thus be helpful for increasing the roll stiffness without affecting vertical stiffness property.
- For vehicles with even load distribution or greater load on the front axle, an appropriate negative PM tuning could considerably improve vertical ride with only slight increase in pitch acceleration response.
- The suspension tuning could help reduce DLC due to forces developed by tires at one of the axles with an increase in DLC due to the other axle tire forces. A negative PM tuning tends to deteriorate the DLC responses of vehicles, irrespective of the load distribution and road roughness. A positive tuning, however, could generally yield slightly lower DLC.
- The suspension tuning could improve the rms travel response of suspension at one of the axles, while that of the suspension at the other axle would generally deteriorate. Suspension tuning with a positive PM tends to slightly deteriorate suspension travel responses of the vehicles with even or more load on the rear axle. A negative PM tuning, however, could slightly improve

suspension travel responses of the vehicles with even or more load on the rear axle.

- Unlike unconnected suspension, pitch plane coupled suspension, could permit independent tunings of the vertical and pitch stiffness rates, and thus the PM and PSR. A pitch-connected suspension could thus offer significant potential for improving both the ride and handling qualities of vehicles.

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